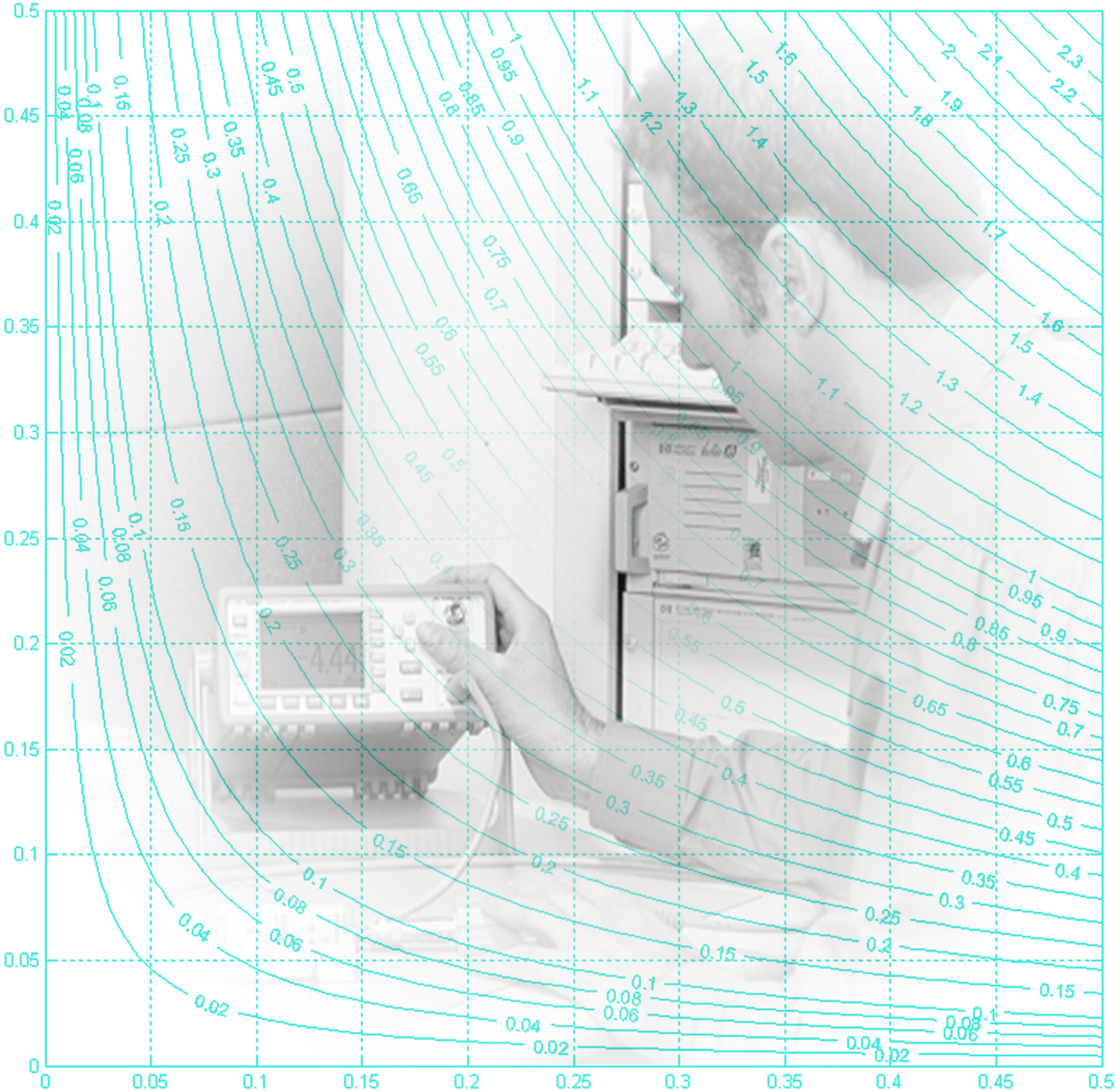
Solutions for Minimizing Measurement Uncertainty and Quick and Easy Estimation of Uncertainty Due to Mismatch

Application Note



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1.0 Introduction

All measurements are subject to uncertainty, whether from the instrumentation being used to make the measurement, the item being measured, the engineer conducting the measurement, or the measurement environment. Perhaps one of the largest contributors to the total uncertainty for RF and microwave power measurements is mismatch uncertainty. It arises from an incomplete knowledge of the phase of the reflection coefficients of the source and load impedances, plus their interconnection. Because the mismatch term almost always predominates, it requires extra attention from the engineer who can use simple procedures to minimize its effect.

Instrumentation (e.g., a spectrum/signal analyzer or power meter) also contributes significantly to measurement uncertainty. The magnitude of this uncertainty and the factors responsible for it depend on the instrument in question. In a signal analyzer, for example, sources of uncertainty include frequency response, input attenuator setting, resolution bandwidth switching, and the calibrator.

Regardless of where the measurement uncertainty stems from, the end result is an impact on measurement accuracy and certainty; the higher the uncertainty, the lower the engineer's confidence in a given measurement. The goal, therefore, is to be able to quickly and accurately calculate measurement uncertainty, and minimize it where ever possible.

This application note presents a number of techniques that can be used to minimize mismatch uncertainty. It also presents techniques for combining measurement uncertainties, and in particular, models that can be used to determine mismatch uncertainty when phase is not known. One such model, the Rayleigh model, leads to substantially more accurate, yet still conservative, estimates of standard uncertainty due to mismatch, as compared to the methods commonly used. In fact, the Rayleigh-based process typically results in a six times lower estimate of uncertainty than the popular U-shaped distribution method.

2.0 Measurement Challenge: Estimating Measurement Uncertainty

Estimating and minimizing measurement uncertainly can be a daunting task, one that's critical for allowing engineers to apply a smaller guardband. By definition, a guardband is the difference between the acceptance test limit and performance limit (specification). It accounts for measurement uncertainties, as well as changes in performance due to external conditions, drift and any other mechanism that may affect performance. The application of a guardband ensures, with a high level of confidence, that a product measured and found to be within the test limit will meet specification.

To apply a smaller guardband and, in turn, increase measurement confidence and yield, measurement uncertainty must be accurately estimated. While various techniques can be used to accomplish this task (e.g., more averaging or lower IF resolution bandwidth), such methods often come with undesirable consequences like longer measurement time. Today's engineers require a simpler, quicker way of estimating measurement uncertainty, and in particular, mismatch uncertainty—one that is conservative enough to enable the use of a smaller guardband.

3.0 Addressing Mismatch Uncertainty

A number of simple and advanced techniques can now be employed to minimize mismatch uncertainty. To better understand these techniques first consider that mismatch uncertainty, or mismatch loss uncertainty, is defined as the amount of power (expressed in dB) unavailable on the output of a transmission line due to signal reflections and impedance mismatches. If properly terminated, the transmission line has no reflections and therefore, no mismatch loss.

Here, Γ_l is the reflection coefficient and is frequently expressed in terms of its magnitude, ρ_l, and phase, ϕ_l. Γg is the reflection coefficient looking back into a generator attached to a power meter (sensor) and is express in terms of its magnitude, ρ_g, and phase, ϕ_g. Γ_l and Γg are seldom completely known for both magnitude and phase. Only the magnitudes ρ_l and ρ_g are usually measured or specified. This lack of phase information makes it is impossible to exactly calculate the net power delivered by the generator to a load P_{gl} and the ratio of the maximum available power, P_{av} to P_{gl}. Minimum and maximum values, however, can be found.

The maximum and minimum values of 10 log$|1 - \Gamma g \Gamma_l|^2$ are called mismatch loss uncertainty limits (M_u). The maximum occurs when $\Gamma g \Gamma_l$ combines with "one" in phase to yield:

$$M_{u\ max} = 10 \log (1 + \rho_g \rho_\ell)^2 \qquad \text{Equation 1}$$

This maximum limit is always a positive number and cannot be larger than 6 dB, which occurs when $\rho l = \rho_g = 1$. The minimum value of the mismatch loss uncertainty occurs when $\Gamma g \Gamma_l$ combines with "one" exactly out of phase to yield:

$$M_{u\ min} = 10 \log (1 - \rho_g \rho_\ell)^2 \qquad \text{Equation 2}$$

The minimum limit is always a negative number. Its magnitude is greater than the magnitude of the maximum limit, but usually by a very small amount.

Note that mismatch loss uncertainty limits can also be specified as a percent deviation from "one" rather than in dB. This is given by:

$$\%M_u = 100\, [(1 \pm \rho_g \rho_\ell)^2 - 1] \qquad \text{Equation 3}$$

For mismatches less than 2 percent, the following approximation can be used:

$$M_u > \pm\, 200\, \rho_g \rho_\ell\, \% \qquad \text{Equation 4}$$

Modern engineering electronic calculators have programs available for calculating mismatch loss uncertainty limits, either in terms of standing wave ratio (SWR) or ρ. Computer-aided engineering models often contain routines for such transmission line calculations.

3.1 Simple techniques for reducing mismatch uncertainty

A number of practical techniques can be employed to control mismatch uncertainty. The first involves selecting test equipment with the lowest SWR specification. In this case, controlling mismatch uncertainty is as simple as reducing the reflection coefficient on any transmission lines or components that are part of the test arrangement. Other steps that can be taken to ensure test system performance does not become degraded include:

- Minimize cable length and number of adapters. At lower frequencies (e.g., lower than 300 MHz) the length of the transmission lines should be minimized to reduce the changes of phase with frequency. For higher frequencies this method is not viable, because even short lengths of cable form significant fractions of a wavelength. Good quality cable should also be used. If testing multiple devices, the connectors should be designed for hundreds of connection/disconnection cycles.

  Additionally, using adapters to convert between different families of connectors may be unavoidable, but should be minimized. Adapters should convert directly and not be stacked. Also, be wary of mating between dissimilar connectors. For example, APC-3.5 and SMA look very similar but have different mechanical interfaces. The use of a precision adapter or "connection saver" is recommended between APC3.5 and SMA connectors.

- Use a torque wrench for consistency and apply appropriate torque values. When tightening screw-type connectors, use a torque wrench to avoid over- or under-tightening the connector. This will ensure there is little variation in tightness when another operator takes over.

- Characterize cables, connectors, and adapters. The best way to check the performance of cables and adapters is to use a vector network analyzer and record the results for comparison at the next regular test station audit. The best way to ensure the performance of precision connectors is to clean and gauge them regularly. When a connector is gauged, it is measured with a special dial gauge to ensure it has not been mechanically damaged. A damaged connector can instantly ruin the mated part.

3.2 Advanced techniques for improving mismatch uncertainty

When the performance of a test arrangement is simply not good enough for the job, a number of more advanced techniques can be employed to improve mismatch uncertainty and in turn, accuracy. These techniques include:

- Add an attenuator to one end of the transmission line to improve the test SWR. The use of an attenuator (pad) to improve the flatness of a transmission line assumes that the return loss of the attenuator is better than the original source or load. The attenuator is usually placed at the end of the line with the worst return loss. To keep the signal level constant at the load, the generator level must be increased, although doing so limits the applicability of this method to the mid-range of power levels.

- Use an isolator component to reduce reflections from a load. Isolators are applied at high power levels, where the economic cost of the power lost in an attenuator would be high, and at very low power levels, where the signal would be masked by thermal noise. Isolator components are narrowband devices and are likely to be more expensive than attenuators.

- Use the power-splitter method. Here, a leveling loop is employed to create a Z_o impedance at the centerpoint of the splitter. The resulting "generator output impedance" is equivalent to the highly-matched microwave resistor in the second arm of the splitter. The leveling loop uses low-frequency feedback to improve the effective source match to the line. This requires a two-resistor power splitter or a directional coupler. The output of the generator is measured on a power meter and the generator is adjusted so that the indicated power is at the level needed. The technique requires a power meter that is better matched than the signal generator, and an accurately matched two-resistor power-splitter or directional coupler.

As the measurement frequency increases, so too does the importance of maintaining a low SWR on the transmission line. While mismatch uncertainty can never be completely eliminated, these practical measures will allow the engineer to keep SWRs to a minimum.

4.0 Instrument Uncertainties

Instrument uncertainty is another contributing factor in overall measurement uncertainty. Two instruments that are often used in RF and microwave power measurements are the signal analyzer and the power meter. Each has its own unique sources of uncertainty.

4.1 Signal analyzer

Figure 1 shows a simplified block diagram of the elements in a typical signal analyzer, some of which contribute uncertainty to amplitude measurements. These contributions or amplitude uncertainty factors are specified for most signal analyzers and listed in Table 1. The range of values for each factor covers a variety of signal analyzers. Most signal analyzers have specifications for both absolute and relative uncertainties.

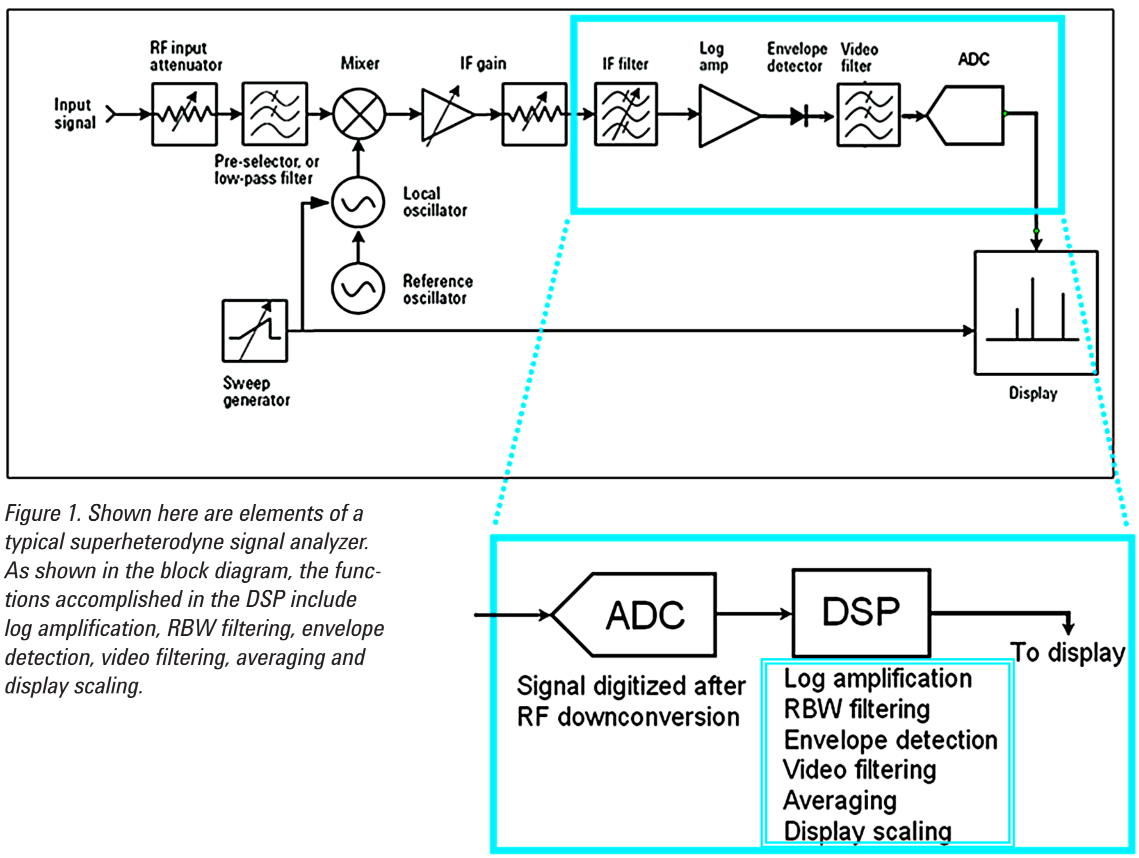

Figure 1. Shown here are elements of a typical superheterodyne signal analyzer. As shown in the block diagram, the functions accomplished in the DSP include log amplification, RBW filtering, envelope detection, video filtering, averaging and display scaling.

Relative uncertainties affect both relative and absolute measurement accuracy. Some of the factors affecting relative measurement uncertainty are:

- Frequency Response (Flatness)—Signal analyzer frequency response is often the single highest contributor to uncertainty and is a function of input attenuator flatness, mixer conversion loss and preselector flatness (if applicable). It is frequency-range dependent, specified as ±n dB over a frequency range at a given attenuator setting, the frequency response affects the displayed amplitudes of signals at different frequencies. It is also usually specified for both relative and absolute measurements.

  Relative frequency response uncertainty describes the largest possible amplitude uncertainty over a given frequency range relative to the midpoint between the amplitude response extremes within that frequency range. The relative frequency response specification for a given frequency range tends to be lower than the absolute frequency response specification over the same range. To obtain the frequency response uncertainty for relative amplitude measurements within a band, the relative frequency response specification must be doubled to reflect the peak-to-peak frequency response (often greater than the absolute frequency response spec.). Some signal analyzers require engineers to "peak" the preselector for frequency response specifications to be valid.

  A low-frequency RF analyzer might have a relative frequency response uncertainty of ±0.5 dB. A microwave signal analyzer tuning in the 20 GHz range could well have an uncertainty in excess of ±4 dB.

- Band Switching—Mixing the input signal with harmonics of the local oscillator (LO) allows measurements over a very wide frequency range. Each LO harmonic provides a different harmonic frequency band within the analyzer's overall frequency range. When signals in different harmonic bands are measured, additional uncertainties arise as the analyzer switches from one band to another. On some signal analyzers, the band-switching points are visible as discontinuities in the displayed noise floor. The engineer can verify whether or not their measurement involves more than one band by referring to the analyzer's specifications for the frequency ranges related to each harmonic band. Although not always specified, a typical band switching uncertainty is ±1 dB.

Table 1. Amplitude uncertainty factors

Relative	± dB
Frequency response (flatness)	0.5 to 4
Band switching	0.5 to 1
Scale fidelity	0.5 to 2
Reference level (IF gain)	0.1 to 1
input attenuator switching	0.5 to 2
Resolution bandwidth switching	0.1 to 1
Display scale switching	0.0 to 1
Absolute	**± dB**
Frequency response	0.5 to 4
Calibrator	0.2 to 1

- Input Attenuator Switching—Input attenuation has inherent uncertainty that reduces reference level accuracy, but only when the attenuator setting changes between reference level calibration and measurement. Because the input attenuator operates over the entire frequency range of the analyzer, its step accuracy is a function of frequency. At low frequencies, the attenuator is quite good. At 20 GHz, it's not as good. A typical input attenuator switching uncertainty is ±1 dB.

- Resolution Bandwidth Switching—Different resolution bandwidth (RBW) settings have different insertion loss characteristics which can cause amplitude changes when the same signal is measured with different settings. Changing the bandwidth setting between amplitude measurements degrades accuracy. A typical RBW switching uncertainty is ±0.4 dB.

Absolute measurements are made relative to a calibration signal with known amplitude. Most signal analyzers have a built-in calibrator that provides a signal with specified amplitude at one frequency. A typical calibrator has an uncertainty of ±0.3 dB. The calibrator provides absolute calibration for the top line of the graticule. Since the calibrator source typically operates at a single frequency, the relative accuracy of the analyzer is used to translate the absolute calibration to other frequencies and amplitudes.

If the signal being measured is at a different frequency than the calibrator, the frequency control must be changed. If the signal is at different amplitude, the reference level should also be changed to bring the signal to the top graticule line for best accuracy (if IF gain uncertainty is less than scale uncertainty). Such changes contribute relative amplitude uncertainty to the measurement.

4.2 Power meter

Power meters (including sensors) are often used when measuring RF and microwave power. Some of the factors affecting measurement uncertainty in power meters include:

- Power Meter Calibration—A number of standards and processes are involved in calibration of a power meter. Each can be influenced by various factors like personnel carrying out the calibration or environmental factors such as temperature and humidity that contribute to measurement uncertainty.

- Power Meter—Instrumentation uncertainty is the result of a combination of factors such as meter tracking errors, circuit nonlinearities, range-changing attenuator inaccuracy, and amplifier gain errors. It also includes very small sources of uncertainty arising from things like the thermoelectric voltage introduced by temperature gradients within the electronic circuits and interconnecting cables or the operator's interpretation of the meter indication. This accumulated uncertainty is guaranteed by the instrument manufacturer to be within a certain limit.

- Power Meter Drift—Drift, or long-term stability, is the change in meter indication over a long time (usually one hour) for a constant input power, temperature and line voltage, and is mostly sensor induced. In most cases, drift is actually a drift in the zero setting. For measurements on the upper ranges, drift contributes a very small amount to total uncertainty. On the more sensitive ranges, it can be reduced to a negligible level by zero setting immediately prior to making a reading.

- Sensor Calibration Factor—The calibration factor (K_b) is a combination of the power sensor's effective efficiency and mismatch loss. Accurate measurement of K_b is quite involved and performed mainly by standards laboratories and manufacturers. Most modern power meters can correct their meter reading by setting a dial or keying in a digital number to the proper K_b value. The uncertainty of K_b (stemming from inaccuracies in measurement of K_b by the manufacturer, NIST or standards laboratories) is specified by the calibration supplier.

- Power Sensor Linearity—Power measurement linearity is mostly a characteristic of the sensor. Deviation from perfect linearity usually occurs in the sensor's higher power range or near the sensor's specified limits. For thermocouple sensors, linearity is negligible except for the top power range of +10 to +20 dBm, where the deviation is specified at ±3 percent.

- Power Meter Zero Set—In any power measurement, the meter must initially be set to zero with no RF power applied to the sensor. This is accomplished within the power meter by introducing an offset voltage that forces the meter to read zero. The offset voltage is contaminated by several sources including sensor and circuit noise. On higher power ranges, error in zero setting is small in comparison to the signal being measured.

- Power Meter and Sensor Noise—Noise (short-term stability), arises from sources within the power sensor and circuitry and is specified as the change in meter indication over a short time interval (usually one minute) for a constant input power, temperature and line voltage. One cause is the random motion of free electrons due to the finite temperature of the components. The power observation might be made at a time when this random fluctuation produces a maximum indication, or perhaps a minimum.

5.0 Combining Measurement Uncertainties

While various individual uncertainties from the signal analyzer and power meter have been discussed, at some point total uncertainty must be calculated. This can be accomplished using traditional analysis methods like worst-case or Root Sum of Squares (RSS), or using a Rayleigh distribution model, which provides a more accurate, but still conservative estimate of uncertainty due to mismatch.

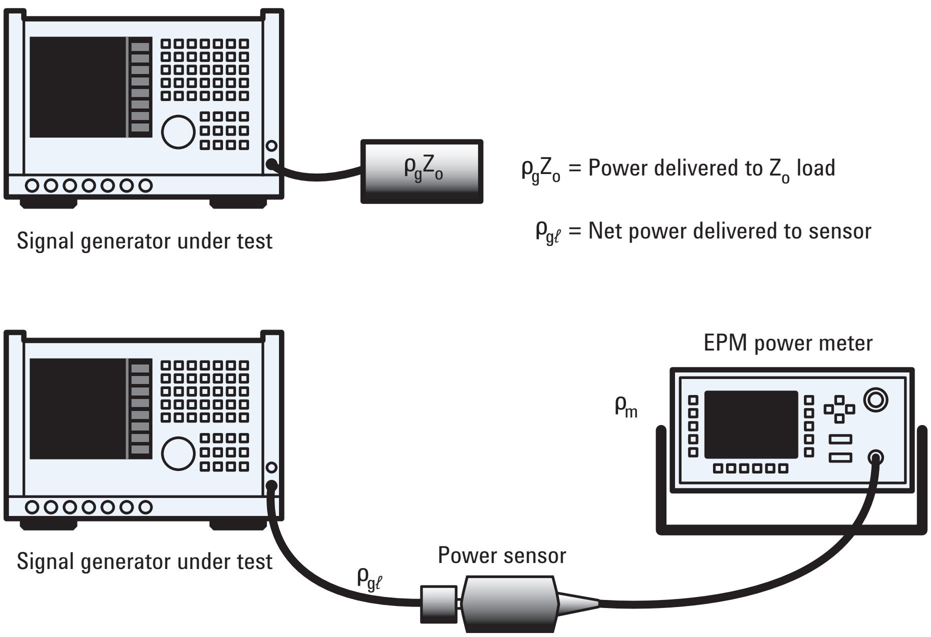

Figure 2. Desired power output to be measured is P_{gzo}, but measurement results in the reading P_m.

To better understand these different methods first consider the image in Figure 2, which is used to develop an equation showing how a power meter reading, P_m, is related to the power a generator would deliver to a Z_o load, P_{gzo}. The equation shows how the individual uncertainties contribute to the difference between P_m and P_{gzo}. The power measurement equation is given by:

$$Pg_{zo} = \frac{M_u(P_m - t)}{K_b m} \qquad \text{Equation 5}$$

where P_{gzo} is the net power delivered to the sensor, M_u is the mismatch uncertainty term, t is the total offset uncertainty, K_b is the calibration factors, and m is the magnification.

In the ideal measurement situation, M_u and K_{bm} are each one, and t is zero. Under ideal conditions, meter reading P_m gives the proper value of P_{gzo}.

5.1 Worst-case uncertainty

One method of combining uncertainties for power measurements in a worst-case manner is to add them linearly. This occurs when all possible sources of error are at their extreme values and in such a direction as to add together constructively, achieving the maximum possible deviation between P_m and P_{gzo}.

As an example, consider the measurement conditions listed at the top of Table 2, which is a chart of the various error terms for the power measurement of Figure 2. The conditions and uncertainties listed in the Table are typical and the calculations are for illustration only as they do not indicate what's possible using the most accurate technique. Calculations are carried out to four decimal places because of calculation difficulties with several numbers of almost the same size.

Table 2. Chart of uncertainties for a typical absolute power measurement. Most errors listed are from a manufacturer's data sheet.

Measurement condition	P_m = 50 µW	Full scale (F.S.) = 100 µW		
	$\rho_\ell \le 0.091$ (SWR ≤ 1.2)	$\rho_g \le 0.2$ ($SWR_g \le 1.5$)		
	K_b = 93% ± 3% (worst case), ±1.5% (RSS)			
Error	Description	Worst case values		RSS component
		P_{gZo} max	P_{gZo} min	$(\Delta X/X)^2$
Mu	$(1 \pm P_g P_\ell)^2$	1.0367	0.9639	$(0.0367)^2$
Kb uncertainty	±3% (w.c.), ±1.5% (RSS)	1.03	0.97	$(0.015)^2$
Components of m				
Ref. osc. unc.	±0.6% (use 2-yr 25 ±10 °C value)	1.006	0.994	$(0.006)^2$
Ref. osc. Mu	SWR_g = 1.05, SWR_ℓ = 1.1	1.002	0.998	$(0.002)^2$
Instrumentation	±0.5% of F.S.	1.01	0.99	$(0.01)^2$
Total m		1.018	0.982	
Components of t				
Zero set	±0.5% F.S. (low range)	+0.05 µW	–0.05 µW	$(0.001)^2$
Zero carryover	±0.2% of F.S	+0.2 µW	–0.2 µW	$(0.004)^2$
Noise	±0.025 µW	+0.025 µW	–0.025 µW	$(0.0005)^2$
Total t		+0.275 µW	–0.275 µW	
Expressions of total uncertainty				
P_{gzo} max	$Pg_{zo\ max} = \frac{M_{u\ max}(P_m - t_{min})}{K_{b\ min}\, m_{min}}$	54.7170 µW		
P_{gzo} min	$Pg_{zo\ min} = \frac{M_{u\ min}(P_m - t_{max})}{K_{b\ max}\, m_{max}}$		45.7111 µW	
ΔP_{gzo}		4.7170 µW	–4.2889 µW	
ΔP_{gzo} /Pm		+9.43%	–8.58%	$(0.001729)^{1/2}$
				±4.2%
				+0.1769 dB
Uncertainty in dB		0.3915 dB	–0.3895 dB	–0.1844 dB

Instrumentation uncertainty, i, is frequently specified in percent of full scale (P_{fs}). The contribution to magnification uncertainty, m, is:

$$m_i = \frac{(1 + i)\, P_{fs}}{P_m} \qquad \text{Equation 6}$$

The uncertainties that contribute to the total magnification uncertainty combine like the gain of amplifiers in cascade. The minimum possible value of m occurs when each of its contributors is a minimum. The minimum value of m (0.9762) is the product of the individual factors (0.988 * 0.998 * 0.99). The factors that contribute to the total offset uncertainty, t, combine like voltage generators in series; that is, they add. Once t is found, the contribution in dB can be calculated using:

$$t_{dB} = 10 \log \left(1 \pm \frac{t}{P_m}\right) \qquad \text{Equation 7}$$

The maximum and minimum values of Pg_{zo} can be calculated using Equation 5, given the values of Table 2. In this case, Pg_{zo} max is 1.0943 P_m and Pg_{zo} min is 0.9142 P_m. Note that the uncertainty in Pg_{zo} may also be stated as an absolute differential in power, a fractional deviation, a percent of the meter reading, or a dB deviation from the meter reading (which can be found by summing the individual error factors expressed in dB).

Figure 3 graphically depicts the contributions to worst-case uncertainty, with mismatch uncertainty as the largest single component of total uncertainty. This is typical of most power measurements. Magnification and offset uncertainties, the easiest to evaluate from specifications and often the only uncertainties evaluated, contribute to less than one-third of the total uncertainty.

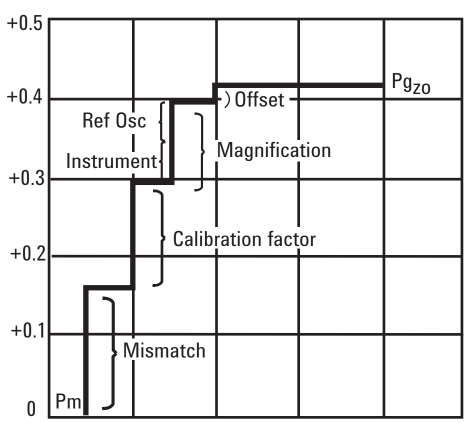

Figure 3. Graph of individual contributions to the total worst-case uncertainty.

5.2 RSS uncertainty method

Whereas the worst-case uncertainty is a very conservative approach, the RSS method offers a more realistic means of combining uncertainties. It is based on the fact that most of the power measurement errors, although systematic and not random, are independent of each other and therefore, their individual uncertainties can be combined in an RSS manner.

Finding the RSS uncertainty requires that each individual uncertainty be expressed in fractional form. The RSS uncertainty for the power measurement (Equation 5) is given by:

$$\frac{\Delta Pg_{zo}}{Pg_{zo}} = \left[\left(\frac{\Delta M_u}{M_u}\right)^2 + \left(\frac{\Delta K_b}{K_b}\right)^2 + \left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta t}{P_m}\right)^2\right]^{1/2} \qquad \text{Equation 8}$$

If not known directly, each of these factors may be found by taking the RSS of several components using:

$$\frac{\Delta M_1}{m} = \left[\left(\frac{\Delta M_1}{m_1}\right)^2 + \left(\frac{\Delta m_2}{m_2}\right)^2 + \cdots\right]^{1/2} \qquad \text{Equation 9}$$

Here, m_1, m_2, and so forth are the reference oscillator uncertainty, the instrumentation uncertainty, and other terms of Table 2. The extreme right hand column of Table 2 shows the components used to find the total RSS uncertainty. The result is ±4.3 percent, which is much less than the worst-case uncertainty of +10.1 percent and –9.1 percent. One characteristic of the RSS method is that the final result is always larger than the largest single component of uncertainty.

5.3 Uncertainty for mismatch model

When the reflection coefficient is known, both magnitude and phase, it is possible to correct for mismatch with known uncertainty. When phase is not known, determining mismatch uncertainty requires a different model. The standard uncertainty of the mismatch expression, $u(M_u)$, assuming no knowledge of the phase, depends on the statistical distribution that best represents the moduli of reflection coefficients Γ_g and Γ_l.

The power dissipated in a load when Γ_l is not 0 is:

$$P_d = Pg_{zo} \frac{1 - |\Gamma_\ell|^2}{|1 - \Gamma_g \Gamma_\ell|^2} \qquad \text{Equation 10}$$

In this case, the numerator is known as mismatch loss, while the denominator represents mismatch uncertainty, M_u, the gain or loss due to multiple reflections between the generator and the load. If both the moduli and phase angles of Γ_g and Γ_l are known, M_u can be precisely determined. Generally, an estimate of the moduli exists, but the phase angles of Γ_g and Γ_l are not known.

When the reflection coefficients of the generator and load are not known, engineers may estimate probabilities of mismatch uncertainty according to the six test cases in Table 3.

Table 3. Summary of test cases used to estimate mismatch uncertainty probability.

Case	$u(M_u)$	Distribution	Type
A	$\frac{1}{\sqrt{2}} \times \lvert\Gamma max_g\rvert \times \lvert\Gamma max_\ell\rvert$	–	Disk/Disk (uniform inside circle)
B	$\sqrt{2} \times \lvert\Gamma_g\rvert \times \lvert\Gamma_\ell\rvert$	U-shape	Ring/Ring (fixed)
C	$\frac{\sqrt{2}}{\ln(20)} \times \Gamma 95_g \times \Gamma 95_\ell$	Bell-shape	Rayleigh/Rayleigh
D	$\lvert\Gamma max_g\rvert \times \lvert\Gamma_\ell\rvert$	–	Disk/Ring
E	$\sqrt{\frac{2}{\ln(20)}} \times \Gamma 95_1 \times \Gamma_2$	–	Ring/Rayleigh
F	Use ISO GUM equation	–	–

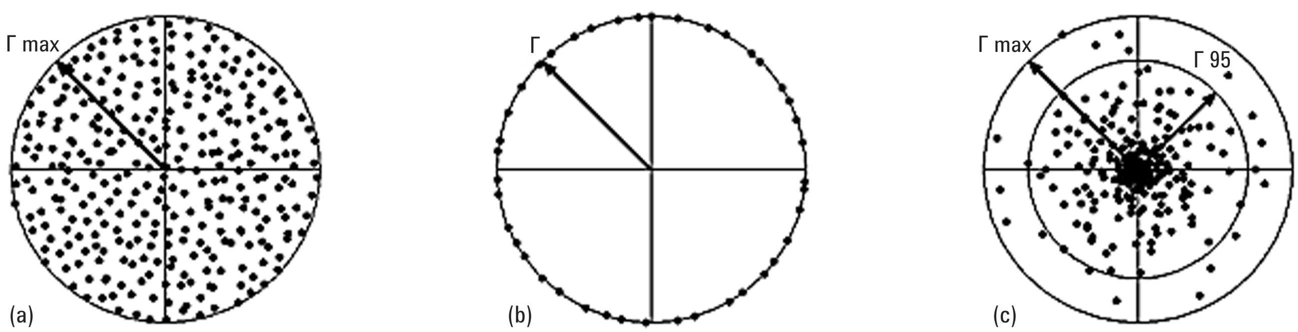

Figure 4. When the reflection coefficients of the generator and load are not known, probabilities of mismatch uncertainty can be estimated using three models: one that uses measured values of reflection coefficient magnitude, another relying on Rayleigh distributed values when reflection coefficient magnitude must be assumed, and the third combining measured and assumed values.

- **Case A:** Two disks distribution of Γ (also known as uniform inside circle)
  Consider Figure 4(a) where both the generator and load have Γ of known maximum specified value of magnitude. For each component, the vector value of Γ has equal probability of lying anywhere within the circle bounded by that maximum magnitude. This distribution has an absolute phase that is uniformly distributed, as well as the relative phase between the two components. The probability distribution function (PDF) of the mismatch uncertainty from this combination is illustrated by the histogram in Figure 5. A closed-form evaluation of the standard deviation (standard uncertainty) of this distribution gives this equation:

$$u(M_u) = \frac{1}{\sqrt{2}} \times |\Gamma\, max_g| \times |\Gamma\, max_\ell| \qquad \text{Equation 11}$$

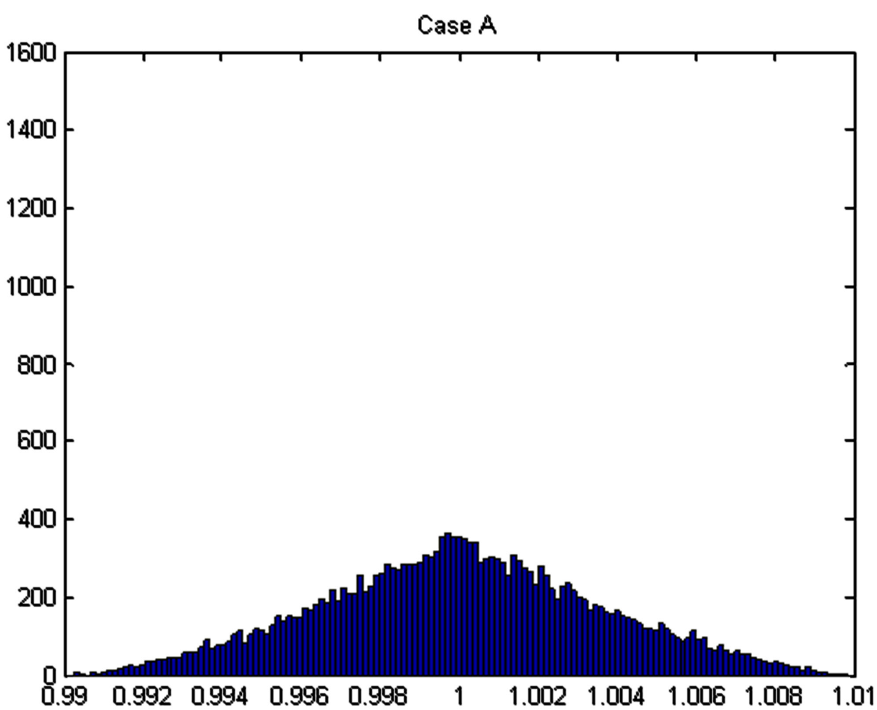

Figure 5. Histogram of M_u for case A, created from a Monte Carlo simulation for Γ_1 max = 0.1 and Γ_2 max = 0.05.

- **Case B:** Two rings distribution of Γ (also known as fixed)
  Consider Figure 4(b) where both the generator and load have Γ of known magnitude. Both the generator and load have an absolute phase that is uniformly distributed, as well as the relative phase between the two components. The PDF of the mismatch uncertainty from this combination is illustrated by the histogram in Figure 6, the well-known U-shaped distribution. A closed-form evaluation of the standard deviation (standard uncertainty) of this distribution gives this equation:

$$u(M_u) = \sqrt{2} \times |\Gamma_g| \times |\Gamma_\ell| \qquad \text{Equation 12}$$

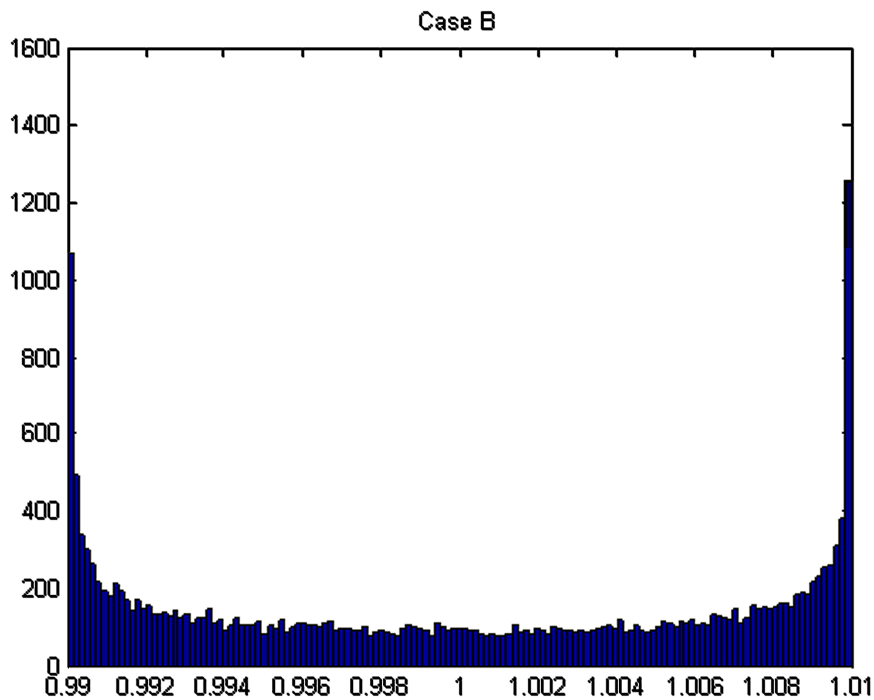

Figure 6. Histogram of Mu for case B, two rings distribution, created from a Monte Carlo simulation for $\Gamma_1 = 0.1$ and $\Gamma_2 = 0.05$.

- **Case C:** Two Rayleigh distribution of $|\Gamma|$

  Consider Figure 4(c) where both Γ have Rayleigh magnitude distributions, with a known 95th percentile, Γ95. The PDF of this case is shown in Figure 7. The mismatch standard uncertainty is:

$$u(M_u) = \frac{\sqrt{2}}{\ln(20)} \times \Gamma95_g \times \Gamma95_\ell \qquad \text{Equation 13}$$

Note that Γ95 is less than max. If max is actually the magnitude Γ corresponding to a 0.27% yield loss in manufacturing (equivalent of "3 sigma" performance in a Gaussian-distributed specification), then Γ95 = 0.712 × Γmax. In practice, Γmax usually exceeds Γ95 by a greater ratio than this, making this a conservative estimation method. Typically, Case C will give lower uncertainties than Case A by a factor of three, and lower than Case B by a factor of six or more.

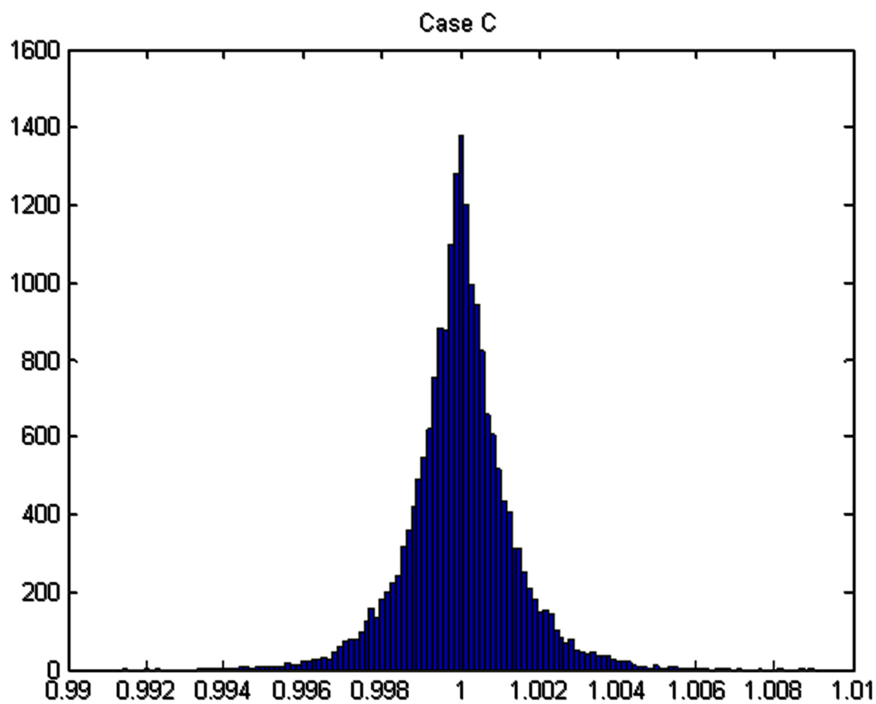

Figure 7. Histogram of M_u for case C, two Rayleigh distribution of magnitude, showing a bell shape.

- **Case D:** One disk and one ring distribution of Γ
  Here, one of the Γ has a known magnitude and the other has a known maximum specified value of magnitude. Both the generator and load have an absolute phase that is uniformly distributed, as well as the relative phase between the two components. The PDF of the mismatch uncertainty from the combination of these two Γ is illustrated by the histogram in Figure 8. A closed-form evaluation of the standard deviation (standard uncertainty) of this distribution gives:

$$u(M_u) = |\Gamma max_g| \times |\Gamma_\ell| \qquad \text{Equation 14}$$

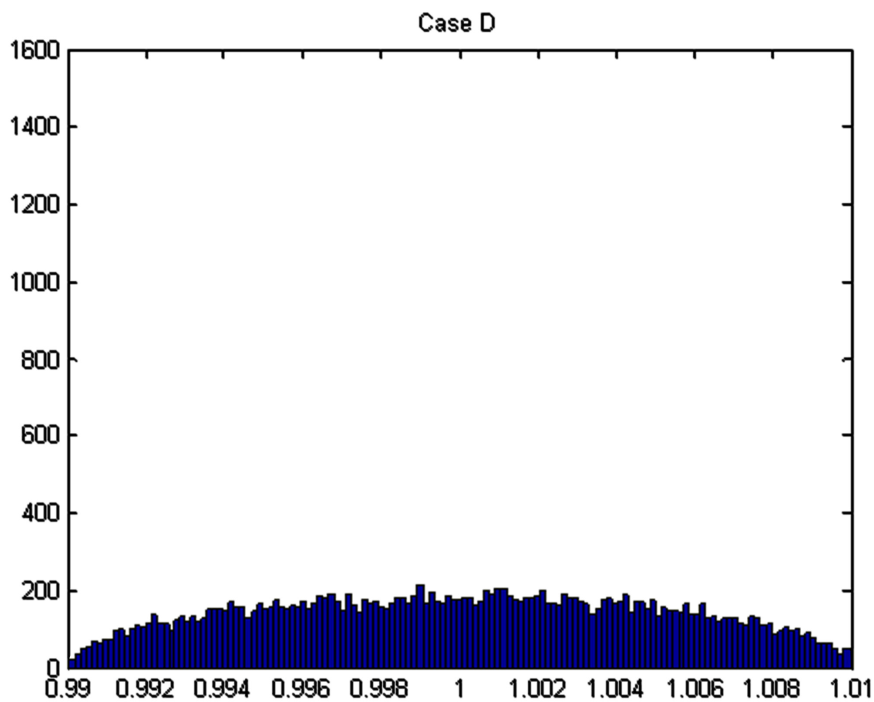

Figure 8. Histogram of M_u for case D, one disk and one ring distribution, created from a Monte Carlo simulation for $\Gamma_1 = 0.1$ and $\Gamma_2 = 0.05$.

- **Case E:** One ring distribution of Γ and one Rayleigh distribution of | Γ|

  Here, one of the Γ has a known magnitude and the other has a known 95th percentile value of magnitude. Both the generator and load have an absolute phase that is uniformly distributed, as well as the relative phase between the two components. The PDF of this mismatch uncertainty is illustrated by the histogram in Figure 9. The shape of this histogram varies substantially with the ratio of the two Γ values. A closed-form evaluation of the standard deviation (standard uncertainty) of this distribution gives:

$$u(M_u) = \sqrt{\frac{2}{\ln(20)}} \times \Gamma 95_1 \times \Gamma_2 \qquad \text{Equation 15}$$

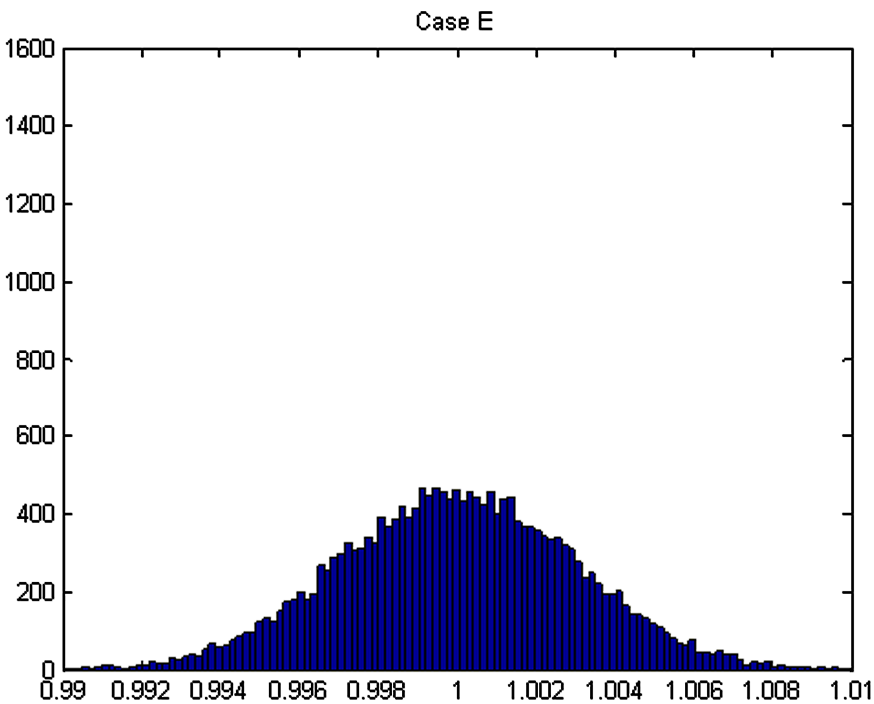

Figure 9. Histogram of M_u for case E, one ring and one Rayleigh distribution, created from a Monte Carlo simulation for $\Gamma_1 = 0.1$ and $\Gamma_2 = 0.05$.

- **Case F:** Known magnitude of Γ, known phase (both generator and load)
  When both and phase are known for the generator and load, the mismatch correction can be applied to the measurement to obtain a more accurate result. In this case, the uncertainty of the mismatch term can be determined using the ISO model (GUM). Here, mismatch standard uncertainty is given by:

$$M_u = |1 - \Gamma_g \Gamma_\ell|^2 \qquad \text{Equation 16}$$

and can be calculated, where $u(M_u)$ is the standard uncertainty, as:

Equation 17

$$u(M_u)^2 = (u(\Gamma_g)\,|1 - \Gamma_g\Gamma_\ell|\,|-\Gamma_\ell|)^2 + (u(\Gamma_\ell)\,|1 - \Gamma g\Gamma_\ell|\,|-\Gamma_g|)^2$$

$$u(M_u) = \sqrt{(u(\Gamma_g)\,|1 - \Gamma_g\Gamma_\ell|\,|\Gamma_\ell|)2 + (u(\Gamma_\ell)\,|1 - \Gamma g\Gamma_\ell|\,|\Gamma_g|)^2}$$

Here, u(Γ) = ρ standard uncertainty of sensor that can be obtained from the measurement report or operating manual.

Analyzing mismatch uncertainty is a complex process. Historically, most engineers have adopted the U-shape distribution method, which results in overreporting of the uncertainty. For engineers looking to minimize the risk of underreporting uncertainty, this can be acceptable. However, the Rayleigh distribution provides a more suitable solution with more realistic values.

6.0 Summary

Calculating uncertainty is a critical and necessary part of any measurement. While instrumentation contributes to total measurement uncertainty, mismatch is perhaps the largest contributor. A number of techniques, both simple and advanced, can be employed to reduce mismatch uncertainty. Analyzing mismatch uncertainty, however, is a complex and time consuming process. While a number of methods can be used to accomplish this task, use of the Rayleigh model provides a quick, accurate estimate of standard uncertainty due to mismatch; one that provides a roughly six times lower estimate of uncertainty than the popular U-shaped distribution method. For today's engineers the benefit of this approach is clear—lower measurement uncertainty means better accuracy and greater confidence in a given measurement.

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